Static Magnetoelastic Coupling in Cubic Crystals, EARL R. CALLEN AND HERBERT B. CALLEN [Phys. Rev. **129**, 578 (1963)]. In the paper, we calculated the equilibrium strains induced by magnetoelastic coupling in a cubic ferromagnet. Both external and internal strain modes were considered. We also calculated the contribution of these strains to the anisotropy energy.

The equilibrium strains were correctly given, despite the fact that the dynamics of the lattice was not explicitly considered; this will be demonstrated below. However, the anisotropy energy was given incorrectly, as the dynamics of the lattice and of the spins are closely correlated.

Let the Hamiltonian of the spin system be H_m, that of a representative strain mode of natural frequency ω be H_ϵ, and the spin-strain interaction be $-B\epsilon\mathcal{K}$. Then

$$H=H_m+H_\epsilon-B\epsilon\mathcal{K}=H_m+\left(\frac{1}{2}c\epsilon^2+\frac{\omega^2}{2c}p_\epsilon^2\right)-B\epsilon\mathcal{K}.$$

The equilibrium strain is given by $\langle\epsilon\rangle=\mathrm{tr}e^{-\beta H}\epsilon/\mathrm{tr}e^{-\beta H}$, and expanding this to first order in the coupling constant B, we find $\langle\epsilon\rangle=(B/c)\langle\mathcal{K}\rangle_0$, which is the value found previously by minimizing the free energy.

The magnetoelastic contribution to the anisotropy energy arises from terms in the free energy quadratic in B. Expanding $-\beta F=\ln \mathrm{tr}e^{-\beta H}$ to second order in B, we find

$$F=F_0-\frac{\hbar\omega B^2}{4c}\int_0^\beta d\beta'\frac{\cosh[(\hbar\omega/2)(\beta-2\beta')]}{\sinh(\beta\hbar\omega/2)}\times\langle\mathcal{K}(-i\hbar\beta')\mathcal{K}\rangle_0,$$

where $\mathcal{K}(-i\hbar\beta')\equiv e^{\beta' H_m}\mathcal{K}e^{-\beta' H_m}$. Even in the classical limit $(\omega\rightarrow 0)$, appropriate to the external strain modes,

$$F=F_0-\frac{B^2}{2\beta c}\int_0^\beta d\beta'\langle\mathcal{K}(-i\hbar\beta')\mathcal{K}\rangle_0,$$

rather than

$$F=F_0-\frac{B^2}{2c}\langle\mathcal{K}\rangle_0^2,$$

as given previously. Both the order of magnitude estimated, and the general symmetry considerations discussed in the original paper remain valid.

Two typographical errors also are to be corrected: In Eq. (35) read $\langle l_1l_2m_1m_2|l_1l_2lm\rangle$ instead of $\langle l_1l_2m_1m_2|l_1l_2l0\rangle$. In Eq. (62a) read $\bar{B}_{0,l}{}^\alpha$ instead of $B_{0,l}{}^\alpha$.

We are indebted to Dr. John Slonczewski, who raised the question of the dynamical effects of the lattice.